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Accounting for Risk Coefficients in Rural Agricultural Producers' Enterprise Choices

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Abstract:

The study looks at the problems of estimating variances and covariances used as measures of risk in designated farm planning tools. The triangular distribution technique which makes use of the range as a measure of variability, is used to generate variance-covariance matrices for a selected representative group of livestock producers; (as an alternative methodology).

A linear programming risk simulation model, which used the variances and covariances generated through the aforementioned procedure, was specified to map out an Income-Risk (E-V) efficiency frontier for representative producers. The results confirm an earlier statement by Brennan (1982) that "the range is equally as efficient as any other sophisticated measure of risk..........".

Introduction:

Over the years, the importance of risk and uncertainty (imperfect knowledge) in the field of decision-making has been the subject of many theoretical discussions and has appeared in the works of many prominent classical theorists notable amongst whom are Bernoulli (1738), Knight (1921), and Hicks (1965). Other studies have also focused on risk as it affects the farmer's practical co-existence with his environment. These include works by Batterham (1971), Driver and Stackhouse (1976), Anderson et. al (1977), Driver (1979), and Turvey (1986).

Risk and uncertainty considerations generally have a significant impact on the way resources should be allocated — a result that raises serious questions about the relevance of any theory of production and producer decision-making that specifically or inadvertently ignores risk

Since imperfect knowledge results in risk and uncertainty in the decision-making process, individuals responding to these situations assume a personal attitude that is reflected in the resource mix and the structural and operational choices for their farm organization (Driver and Stackhouse, 1976). Because risk programming is the most realistic option when dealing with a production process such as farming, ways must be found to help farmers incorporate some risk management measures into their decision-making process.

This study outlines and employs a methodology for specifying and incorporating risk into a linear programming model. Ordinarily, Quadratic Programming (QP) with minos as well as MOTAD are other tools which can be used for modelling risk in the determination of alternative best plans and in the construction of (E-V) income-risk efficiency frontiers.

However, in situations where these tools are not easy to come by or where an alternative method is preferred, a modified linear programming model called LP-RS, which is a linear program incorporating risk as one of its components, could be employed. Developed by Driver and Stackhouse in 1976, it consists of 2 stages: (a)

Linear Programming (LP), and (b) Risk Simulation (RS). Gross margins are discounted by a scalar and a vector of standard deviations of gross margin returns. The scalar takes on values which assume corner point solutions. Hence, as the scalar increases, relative gross margins returns are discounted, the objective function thus changes slope and a new corner point solution is found.

In using either QP with minos, MOTAD or LP-RS, ways must be sought for estimating the risk components. The most reliable method is to solicit for and estimate variances and covariances for each enterprise. However, variances and covariances are measures which are empirically difficult to estimate especially among farmers who do not keep formal records of outputs per technical unit and prices received for output sales through time. Previous researchers have always found it relatively easy to estimate the expected values (means) of gross margins and the variance estimates of gross margins which are the elements on the main diagonal of the variance-covariance matrices. However, the covariances (off-diagonal elements) have always posed more serious estimation problems.

Methodology:

Empirical evidence indicates substantial degrees of correlation between certain farm enterprises (Heady, 1952). To account for this interdependence, an estimate of both the variance and covariances must be made (Bauer, 1971). Ideally, one should construct a subjective joint probability density function involving gross margins of all enterprises to be considered. Through integration, the mean, variance and covariance would be derived.

While this matrix could be established through an interview in the simple case of 2 or 3 enterprises, the task becomes impossible for the decision maker as more activities are added. Farmers often think in terms of worst, best and most likely outcomes but they find it virtually impossible to answer questions concerning enterprise interdependency.

As a result of these problems, other researchers have suggested alternative methods. These include the use of historical price and yield data suggested by Bauer (1971).

The triangular distribution technique as an alternative solution is explored in this paper. The triangular distribution (Trirand Function) generates pseudo-random values following a triangularly shaped distribution. The 3 arguments are the "lower limit", the "most probable" value and the "upper limit" rather than a distribution of values. It is therefore posited that such a distribution would be more easily estimated by farmers who have few records.

In the area of application, its various versions have been used by Hertz (1964), Bauer (1971) and Sonka and Patrick (1984).

Data Collection

Data were collected between October and December 1986 from a representative sample of 80 poultry and 50 hog producers from Oyo, Ondo, Bendel, Kwara and Lagos states of Nigeria with equal representation from each state. The information collected include three year (1983 to 1985) production and price figures. These were supplemented with secondary data from government and extension sources.

The framework used (the Trirand model) was developed to estimate means and the variance-covariance matrices of prices, yields costs and gross margins by soliciting the farmers' expected (most probable) — optimistic (upper values) — pessimistic (lower values) on yields, costs and prices (Table 1).

TABLE 1: PRICE AND YIELD DATA ON A NIGERIAN POULTRY FARM (1985)

(a) Prices and Costs* (₦)

	Lower	Most probable	Upper
Day old chicks	1.00	1.00	1.50
Eggs/dozen (large grade)	2.20	2.50	2.70
Broilers (live, 2.5kg)	7.00	7.50	9.00
Culls	6.00	7.00	8.00
Feed (₦/ton)*	200	240	280
(b) Yield			
Eggs/Layer (annual)	220	240	280
Mortality (rate (%)	10	12	20

The Analytical Techniques:

(i) Estimating the Variance-Covariance Matrices

Recalling that the data consist of "lower", "most probable" and "upper values", the values for prices and yields can be represented as follows:

(1) ....,(price: Lower (P_L), most prob (p_m), Upper (p_m)

(2) ....Yield: Lower (Y_L), most prob (Y_m), Upper (Y_u)

The values so collected were subjected to 2 different treatments viz:

(a) The Monte Carlo Risk Simulation Technique which iterates and simulates a normal distribution from the triangularly distributed data points, generating values for the means, variances and skewness of the data among other things.

Taking egg production as an example, the 3 data points (range) for gross revenue from egg sales in dozens per layer are introduced into the IFPS (monte carlo) model as follows:

(3) ....$GREgg_1$ = Trirand $(P_L Y_L, P_m Y_m, P_u Y_u)$

(4) ....$GREgg_2$ = Trirand $P_L Y_L, P_m Y_m, P_u Y_L)$

where:

Trirand is the triangular distribution function..

$GREgg_1$ is the gross revenue from egg production for any option.

When a specified number of iterations are performed by this function, the mean and standard deviations are computed for all variables specified in the model. The results for a particular farming operation producing hogs twice in the year, January to June hogs (JJP) and July to December hogs (JJD) are shown in Table 2.

(b) The conventional sum of squares technique. Since the data covered a 3-year period, it was possible to generate enough data points to calculate variances and means. The data were processed in the 3 possible ways to produce the following possibilities for gross revenue:

Table 2: MONTE CARLO RISK SIMULATION SOLUTION

FREQUENCY TABLE

PROBABILITY OF VALUE BEING GREATER THAN INDICATED

	90	80	70	60	50	40	30	20	10
GMJJP									
1	92.3	94.1	96.1	97.4	98.6	100.4	102.7	104.4	107.7
GMJDP									
1	84.7	87.3	89.8	91.6	91.5	95.6	98.2	100.9	104.5
GMJJP2									
1	74.5	79.3	83.9	87.7	91.2	95.3	101.1	106.3	114.9
GMDP2									
GMJDP2									
1	66.1	70.5	78.3	83.1	86.8	90.9	97.4	105.6	114.0

SAMPLE STATISTICS

	MEAN	STD DEV	SKEWNESS	KURTOSIS	10PC	CONF	MEAN	90PC
GMJJP2								
1	99.40	8.728	.1	2.8	98.88			99.92
GMJDP2								
1	93.99	10.657	.0	2.7	93.30			94.68
GMJJP								
1	93.04	15.45	.0	2.6	91.64			94.44
GMJDP								
1	88.77	18.22	.1	2.6	87.12			90.42

(i) $P_L Y_L, P_m Y_m, P_u Y_u$

(ii) $P_L Y_u, P_m Y_m, P_u Y_L$

(iii) $P_L Y_L, P_L Y_m, P_L Y_u, P_m Y_m, P_u Y_L; P_u Y_u$

The first option assumes that when yields are high, prices are high and vice-versa, the second assumes that when prices are low it implies that yields are high, the third is made up of all possible cross products (revenues) between prices and yields.

Variances were computed for the 12 farms selected (6 poultry, 6 hogs) using equation 5:

$$(5) \qquad \frac{1}{N-1} \sum_{i=1}^{N} (X_i - \bar{X})^2,$$

where X_i is the gross revenue, with $\bar{X}$ representing its mean and N, the number of observations.

In computing covariances and correlation coefficients, the data points for all farms were pooled together based on the four-group method of analysis used in this study i.e. poultry (large and small)[1] hogs (large and small). These meant that for options (i) and (ii), there were 27 data points for each group; nine from every farm, while for option (iii), there were 81 data points; 27 from each farm.

The crops that are combined with the livestock types here (corn, yam and cassava) are grown under similar climatic conditions, and in most cases are interplanted; thereby exposing them to similar pests and diseases. Given the above, yields are expected to move in the same direction while prices should move in the opposite direction. However, diversification into crop production should offer some measure of negative relationship to an operator's gross margin. Generally speaking therefore, when crop gross margin are low, livestock margins do not have to go down.

Taking into consideration these expected *a priori* relationships between prices and output, the first and third options were not expected to produce realistic variance — covariance matrices which will conform to most of the above expectations. The results for options (i) and (iii) (Tables 4 and 5) confirm this assertion — the two produced variance and covariance matrices with a mixture of positive and negative signs even among crop and livestock types, contrary to expectations. The second option conformed generally to expectations (Table 3). It not only generated variances similar to those generated by the IFPS model, it also produced covariances and Pearson correlation coefficients that conform to *a priori* expectations. Even though the trirand function produced variances similar to those produced by the second op-

[1]For hogs annual production figures between 60 and 120 is classified as small while greater than 120 is classified large. For poultry, between 2000 and 15000, small, while 20,000 and above is large.

tion used under the conventional sum of squares method, the latter method is chosen in this paper because the trirand model produces no covariances and it will be unrealistic to assume zero covariances among all enterprises.

Table 3: VARIANCE-COVARIANCE MARTRIX AND PEARSON CORRELATION COEFFICIENTS (Option ii)

SMALL SCALE SWINE FARMS

COVARIANCE MATRIX

	JJP	JDP	YAM	CASSAVA
JJD	359.00	251.120	-199.124	-475.052
JDP	251.120	362.00	-276.767	-2784.3
YAM	-199.124	-276.767	32105.0	79121.3
CASSAVA	-475.52	-2784.3	79121.3	1440000

PEARSON CORRELATION COEFFICIENTS

	JPP	JDP	YAM	CASSAVA
JJP	1.00000	0.97043	-0.89104	-0.04912
	0.0000	0.0001	0.3000	0.9001
	27	27	3	9
JDP	0.97043	1.00000	-0.92857	-0.20119
	0.0001	0.0000	0.2421	0.6037
	27	27	3	9
YAM	-0.89104	-0.92857	1.00000	0.96682
	0.3000	0.2421	0.0000	0.1644
	3	3	3	3
CASSAVA	-0.04912	-0.20119	0.96682	1.00000
	0.09001	0.6037	0.1644	0.0000
	9	9	3	9

TABLE 4

Table 4: VARIANCE-COVARIANCE MATRIX AND PEARSON CORRELATION COEFFICIENTS (OPTION i)

SMALL SCALE SWINE FARMS

COVARIANCE MATRIX

JJP	550.295	-19.1662	1132:29	-18495.3
JDP	-19.1662	554.653	51182.8	952:592
YAM	1132.29	51182.8	·3142133	-2876135
CASSAVA	-18495.3	952.592	-2876135	2005621

PEARSON CORRELATION COEFFICIENTS

	JJP	JDP	YAM	CASSAVA
JJP	1.00000	-0.035501	0.01134	-0.46554
	0.0000	0.8624	0.9769	0.0515
	27	27	9	18
JDP	-0.03501	1.00000	0.55069	0.02926
	0.8624	0.0000	0.1244	0.9082
	27	27	9	18
YAM	0.01134	0.55069	1.00000	-0.39824
	0.9769	0.1244	0.0000	0.3762
	9	9	9	7
CASSAVA	-0.46554	0.02926	0.39824	1.00000
	0.0515	0.9082	0.3762	0.0000
	18	18	.7	18

Table 5: VARIANCE-COVARIANCE MATRIX AND PEARSON CORRELATION COEFFICIENTS (OPTION iii)

SMALL SCALE SWINE FARMS

COVARIANCE MATRIX

	JPP	JDP	YAM	CASSAVA
JJP	655.491	417.078	-13877.1	-6834.55
JDP	417.078	631.792	15446	1818.71
YAM	13877.1	15446	10353605	-217068
CASSAVA	-6834.55	1818.71	-217068	3076986

PEARSON CORRELATION COEFFICIENTS

	JJP	JDP	YAM	CASSAVA
JJP	1.00000	0.64811	-0.21056	-0.24151
	0.0000	0.0001	0.3869	0.1386
	81	81	19	39
JDP	0.64811	10.00000	0.25458	0.07290
	0.0001	0.0000	0.2929	0.6592
	81	81	19	39
YAM	0.21056	0.2558	1.00000	-0.04922
	0.3869	0.2929	0.0000	0.8564
	19	19	19	16
CASSAVA	-0.24151	0.07290	-0.04992	1.00000
	0.1386	0.6592	0.8564	0.0000
	39	39	16	39

The LP-RS model is represented as

(6) ....Max $(E(C) — KS)\ W_j$

Subject to:

(6a) ....$AW_j \leq B$

(6b) ....$W_j \geq 0$

(6c) ....$E(C)'\ W_j = ENFI$

(7) ....$(W_j\ Q_{Wj})\ 0.5 = SDNFI$

Where

E(C) is a lxn vector of expected revenues and costs

K is a parameterized scalar,

S is a vector of standard deviations associated with the activities,

A is an Mxn matrix of technical coefficients,

B is an Mxn vector of resource levels,

W_j is a vector of real activities,

ENFI is the expected net farm income of the jth strategy from the model,

SDNFI is the standard deviation of ENFI,

$W_j\ QW_j$ is the variance,

Q is an mxn variance-covariance matrix of returns.

(ii) Incorporating Risk Coefficients into the LP Model

Once the estimation of the variances and covariances is accomplished, the next step is to incorporate them into the Linear Programming Model. Where the LP-RS program is available, this is easily done by including the square roots of the diagonal elements (standard deviations of the gross margins) in the data and requesting the program to iteratively discount the production and revenue parameters by multiples of the standard deviations to generate corner point solutions similar to those generated by other tools. (for example Quadratic programming). The procedure generates the maximum expected net farm income for each multiple (K) as shown in equation (6). The variances and covariances could also be used in the application of other farm planning tools.

It is also possible in places where only the Linear program is available to introduce multiples of the standard deviations for each activity in the optimal solutions given by the LP model by hand. A hand or desk calculator can then be used to iteratively generate new corner point solutions which are deficient only in being less continuous and more discrete.

It has been shown by Driver and Stackhouse (1976) that the solutions produced by the LP-RS model are good approximations of the QP solutions since divergence between the 2 solutions occurs only after significant numbers of resources enter the solution as slacks.[2]

As stated earlier, equation (6), the Expected Net Farm Income could either be computed internally by the IBM-MPSX program or manually using a desk calculator. Equation (7), the standard deviation of the Net Farm Income (SDNFI) is also calculated for every computed Net Farm Income. It could be done by using a separate program called APL Risk Calculator or manually; in both cases, by combining the optimal solutions (enterprise levels produced by the LP model) and the Variance-covariance of the enterprises as shown in the equation (7).

The Realistic (Risk-Conscious) Optimal Farm Plan

Given these 2 results i.e. equation (6) and (7), the income-risk (E-V) efficiency loci for every farmer can be generated. An example is shown in Figure 1. In other words, a portfolio analysis of the farmer's enterprise combinations can be carried out which is basically a compilation of a set of Expected Net Returns along with associated risk levels, by varying the farmer's portfolio composition a-la the E-V frontier.

From these, a selection of a portfolio which maximizes income subject to a minimum level of risk the farmer would be willing to face can be determined, thus maximising the farmer's expected utility as a function of income and rel[illegible] risk levels.

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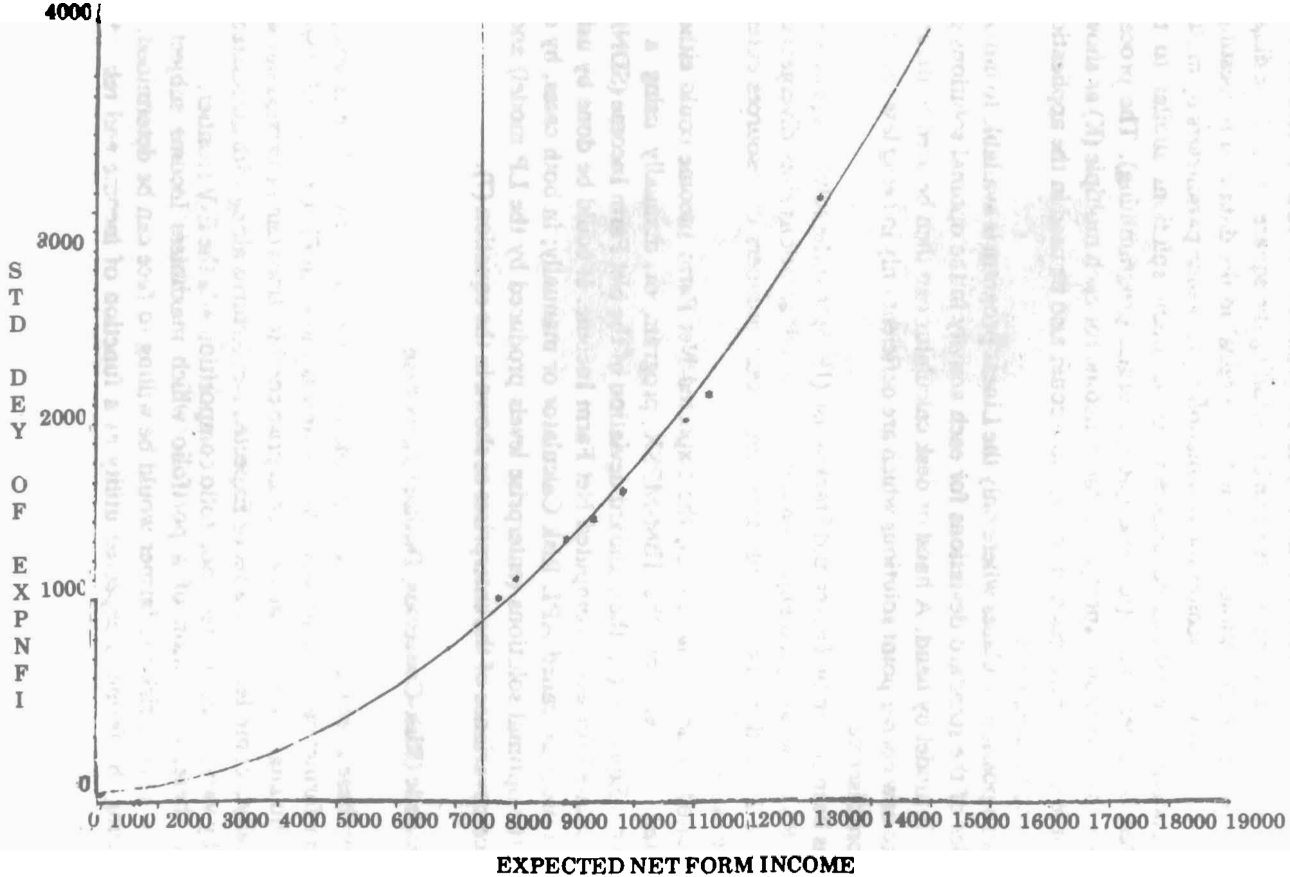
[2]For a better exposition of the comparison of QP with LP-RS solutions refer to Driver and Stackhouse (1976).

FIGURE 1: STATUS QUO E—V EFFICIENCY FRONTIER FOR A NIGERIAN BROILER FARM



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